Image Denoising with Common Vector Elimination based PCA

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Abstract—This paper proposes a novel image denoising technique using Principal Component Analysis (PCA). PCA is a technique useful for classification and compression of data.The purpose of PCA is to reduce dimensionality of data sets or samples by exploring new set of variables smaller than original set of variables and retains most of the samples information.Noise is having some common characteristics and it is uniformly distributed in the image.The noisy image can be decomposed by the PCA into different blocks.Eigen values for each block is calculated and the common vector from each block is eliminated.The denoising algorithm is validated by numerical experiments on grey scale lena image.Numerical results shows that the proposed method can obtain higher PSNR than former methods.

Index Terms – Common Vector ,Eigen values, Image Denoising, Noise , PCA , PSNR, Wavelet Thresholding.

I .INTRODUCTION

Removal of noise is very important area of research. PCA is the most successful technique that has been used or image classification ,recognition as well as compression.PCA reduces large dimensionality of data into smaller dimensionality.PCA can do prediction , redundancy removal ,feature extraction and data compression[1].Wavelet transform has also shown good performance in terms of PSNR in removing additive Gaussian noise. Wavelet based denoising technique has been pioneer in image denoising in past few years[2][3].It is also seen that wavelet does not handles the directional information of natural images. It has been seen that wavelets are not the best choice to sparsely represent natural images[1]. PCA provides an efficient approach for image denoising. PCA resizes blocks of high frequency subbands into vectors and then calculate the eigen values.An image gets corrupted with different types of noise during the processes of transmission, reception, and storage &retrieval. Noise may be classified as substitutive noise and additive white Gaussian noise.

II. RELATED WORK

In this section we present the recent works provides us some useful information for development of the proposed work. A comprehensive review of the literature on image denoising is beyond the scope of this paper.We only present a brief summary of the closest related work.

One of the approach is Non Local Means(NLM) image denoising algorithm that uses PCA to achieve higher accuracy.The author proposes quantitative and qualitative comparision of of NLM and another image neighbourhood PCA based image denoising algorithm[4].The accuracy and computation cost of the NLM image denoising algorithm is improved by calculating neighbouhood similarities after aPCA projection to lower dimensional space.Another paper proposes a method using contourlet transform and 2DPCA. The contourlet transform performs multiresolution and multidirectional decomposition to the image, while 2DPCA is carried out to estimate threshold. Proposed method has a better performance than the classical wavelet soft thresholding[1]. Philip Langley [5] proposes another algorithm for analyzing changes in ECG morphology based on principal component analysis is presented and it is applied to the derivation of surrogate respiratory signals from single-lead ECGs. The respiratory-induced variability of ECG features,P waves and T waves are described by the PCA.The author assessed which ECG features and which principal components yielded the best surrogate for the respiratory signal. Twenty subjects performed controlled breathing for 180 s at 4, 6, 8, 10,12, and 14 breaths per minute and normal breathing. ECG and breathing signals were recorded. Respiration was derived from the ECG by three algorithms: the PCA-based algorithm and two established algorithms, based on RR intervals and QRS amplitudes .ECG-derived respiration was compared to the recorded breathing signal by magnitude squared coherence and cross-correlation. The top ranking algorithm for both coherence and correlation was the PCA algorithm applied to QRS complexes.

Turgay Celik [8] proposes a novel technique for unsupervised change detection in multi temporal satellite images using principal component analysis (PCA) and k-means clustering. The difference image is partitioned into blocks. Ortho normal Eigen vectors are extracted through PCA of h× h non overlapping block set to create an eigenvector space Simulation results show that the proposed algorithm performs quite well on combating both the zero-mean Gaussian noise and the other noise, which is quite attractive for change detection in optical and SAR images.Guangyi Chen[7] proposes a new method for hyperspectral data cubes that have a good signal to noisr ratio(SNR). The author proposes to decorrelate the image information of hyperspectral data cubes from the noise by using PCA. A 2D bivariate wavelet thresholding method is used to remove the noise for low energy PCA channels. S.Sudha, G.R.Suresh and R.Sukanesh [9] presents a wavelet-based thresholding scheme for noise suppression in ultrasound images. Quantitative and qualitative comparisons of the results obtained by the proposed method with the results achieved from the other speckle noise reduction techniques demonstrate its higher performance for speckle reduction. T.Ratha Jeyalakshmi and K.Ramar [10] they described and analyzed an algorithm for cleaning speckle noise in ultrasound medical images. Mathematical Morphological operations are used in this algorithm. This algorithm is based on Morphological Image Cleaning algorithm (MIC). The algorithm uses a different technique for reconstructing the features that are lost while removing the noise. For morphological operations it also uses arbitrary structuring elements suitable for the ultrasound images which have multiplicative noise. Pier rick Coupe's, Pierre Hillier, Charles Kervrann and Christian Barillot [11] proposed a Bayesian Non Local Means-Based Speckle Filtering In their proposal, a new version of the Non Local (NL) Means filter adapted for US images is proposed. Originally developed for Gaussian noise removal, a Bayesian framework is used to adapt the NL means filter for noise. Experiments were carried out on synthetic data sets with different speckle simulations. Nonlocal Means-Based Speckle Filtering for Ultrasound Images is presented by [12] In this method, an adaptation of the nonlocal (NL) means filter is proposed for speckle reduction in ultrasound (US) images. Originally developed for additive white Gaussian noise, we propose to use a Bayesian framework to derive a NL-means filter adapted to a relevant noise model. Results on real images demonstrate that the proposed method is able to preserve accurately edges and structural details of the image. M. I. H. Bhuiyan, M. Omair Ahmad, Fellow, IEEE, and M. N. S. Swamy [13] presented Wavelet-Based Despeckling of Medical Ultrasound Images with The Symmetric Normal Inverse Gaussian Prior In their proposal, an efficient wavelet-based method is proposed for despeckling medical ultrasound images. A simple method is presented for obtaining the parameters of the SNIG prior using local neighbors. Thus, the proposed method is spatially adaptive. Jeny Rajan and M.R. Kaimal [14] In their paper they discuss the speckle reduction in images with the recently proposed Wavelet Embedded Anisotropic Diffusion (WEAD) and Wavelet Embedded Complex Diffusion (WECD). Both WEAD and WECD produce excellent results when compared with the existing speckle reduction filters. Philip Langley proposed a denoising method for hyper spectral data cubes .Experimental results demonstrated that the proposed denoising methods produces better denoising results in terms of PSNR. Ioana Firoiu, Corina Nafornita [18] proposes the use of a recently introduced hyperanalytic WT (HWT), in association with filtering techniques already used with the discrete wavelet transform. The result is a very simple and fast image denoising algorithm. Lei Zhan& Rastislav Lukac [19] proposes a principle component analysis basedspatially-adaptive denoising algorithm, which works directly on Colour Filter Array data using a supporting window to analyze the local image statistics. By exploiting the spatial and spectral correlations existed in the CFA image, the proposed method can effectively suppress noise while preserving color edges and details. Experiments using both simulated and real CFA images indicate that the proposed scheme outperforms many existing approaches, including those sophisticated demosaicking and denoising schemes, in terms of both objective measurement and visual evaluation.

III. Denoising using Principal Component Analysis :

Principle component analysis (PCA) is a technique used for compression and classification of data. The purpose is to reduce the dimensionality for a data set by findig a new set of variables, smaller than the original set of variables and retains most of the sample's information.By information we mean the variation which is present in the sample, given by the correlations between the original variables. The new variables is known as principal components (PCA), are uncorrelated, and are ordered by the fraction of the total information each retains. The noisy image can be decomposed by the PCA into different blocks.Eigen values for each block is calculated and the common vector from each block is eliminated.

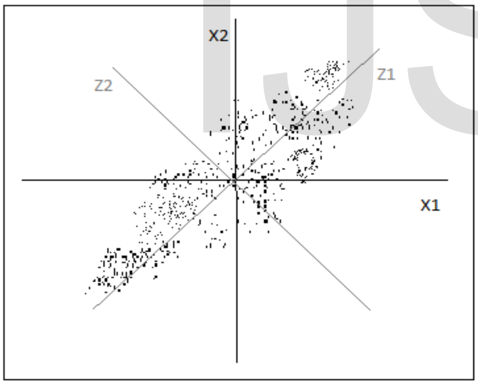

Fig 1. Geometric picture of Principal Components

A sample of n observations in 2D space $X=(x_1, x_2)$

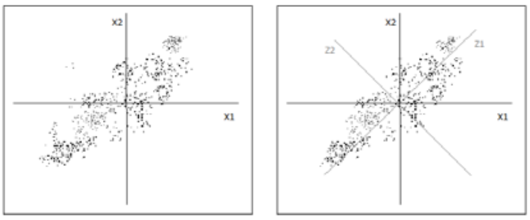

Fig 2. Geometric Picture of new components Z1& Z2.

The 1stPC $Z_1$is a minimum distance from a line in X space

The 2nd PC Z_2 is a minimum distance from a line in the plane perpendicular to the 1s

Let a sample of n observations on vector of p variables

$X=(x_1,x_2,\ldots,x_p)$

The first principal component of the sample by the linear transformation $z_1 \equiv a_1^T x = \sum_{i=1}^{p} a_{i1}\, x_i$ .. (1)

Where the vector$a_1=(a_{11},a_{21},\ldots,a_{p1})$ Is chosen such that $VAR[z_1]$ is maximum

Kth principal component of the sample by the linear transformation

$$z_k \equiv a_1^T x \qquad k = 1, \ldots, p \qquad .(2)$$

Where the vector

$a_k=(a_{1k},a_{2k},\ldots,a_{pk})$

Is chosen such that $var[z_k]$ is maximum subject to $COV[zk,z1] = 0$ for $k > l \geq 1$ (3)

and to $a_k^T a_k = 1$

Calculation of coefficient vectors a_k

$$var[z_1] = \langle z_1^2 \rangle - \langle z_1 \rangle^2 \qquad (4)$$

$$= \sum_{i,j=1}^{p} a_{i1} a_{j1} \langle x_i x_j \rangle - \sum_{i,j=1}^{p} a_{i1} a_{j1} \langle x_i \rangle \langle x_j \rangle \qquad (5)$$

$$= \sum_{i,j=1}^{p} a_{i1} a_{j1} S_{ij}$$

where$S_{ij} \equiv \sigma_{x_i x_j} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$

$$= a_1^T S_{a_1} \qquad (6)$$

S is the convariance matrix for the variables $X=(x_1,x_2,\ldots,x_p)$

To find a_1 maximize $var[z_1]$ subject to $a_1^T a_1 = 1$

Let λ be a Lagrange multiplier

Then maximize $a_1^T S_{a_1} - \lambda(a_1^T a_1 - 1)$

By differentiating... $S_{a_1} - \lambda a_1 = 0$ (7)

$$\Rightarrow (S - \lambda I_p) a_1 = 0 \qquad (8)$$

Therefore a1 is an eigenvector of S corresponding to eigenvalue $\lambda \equiv \lambda_1$

As we have maximized

$\mathrm{Var}[z_1] = a_1^T S_{a_1} = a_1^T \lambda_1 a_1 = \lambda_1$ (9)

So λ_1 is the largest eigenvalue of S

The first PC z_1 retains the greatest amount of variation in the sample.

Sure shrink method is mostly used as orthonormal wavelet transform for wavelet thresholding. The key idea behind SURE shrink is to set to zero all coefficients below a particular threshold value T, while shrinking the remaining ones by this same value; this technique is thus also known as soft thresholding.

$\eta(y) = \mathrm{sign}(y)(|y| - T)+$ (10)

The soft thresholding technique has been shown to be near optimal value. The threshold value T is then selected so as to minimize the risk level. The mean squared error (MSE) in the image domain is preserved in the wavelet domain. Hence, we can write it as follows:

$\mathrm{MSE(Image\ Domain)} = \frac{1}{N}\sum_{i=1}^{N}(\dot{fi} - fi)^2$ (11)

$= \frac{1}{N}\sum_{j=1}^{j}\sum_{i=1}^{N_j}(\ddot{x}_i^j - x_i^j)^2$ (12)

= MSE (Wavelet Domain)

Where N is the number of samples;

J is the number of channels;

NJ is the number of samples in the channel j.

I is the it sample of the jet channel.

As the non-noisy wavelet coefficients are unknown, we need to estimate the MSE using Stein's unbiased risk estimator (SURE).Its minimization according to our particular estimator ˆx = η(y) leads to:

$\mathrm{SURE}_j(t,y) = \sigma^2 - 1/Nj\,(2\sigma^2.\#\{i: |yi| <= t\} + \sum_{i=1}^{Nj}\min(|yi|, t^2)$ (13)

The resulting threshold is thus:

$T_j = \mathrm{argmin}\,(\mathrm{SURE}_j(t,y))$ (14)

Donoho's has proposed the fixed thresholding based reduction of noise in images . Here, the value of threshold (t) is computed as:

$t = \sigma\sqrt{2\log(n)}/n$ (15)

where σ=MAD/0.6745 where MAD is the median of wavelet coefficients and *n* is the total number of wavelet coefficients.

Global Thresholding (wt_q)

This is known as fixed threshold or global thresholding method and it is calculated as:

$wt_q = \sqrt{2\log(n)}$ (16)

where n is the total number of wavelet coefficients.

Rigrsure(wt_{su})

Steins unbiased risk estimator (SURE) or rigrsure is an adaptive thresholding method which is proposed by Donoho and Jonstone .

Heursure (w_{th})

Heursure threshold is a combination of SURE and global thresholding method. If the signal-to noise ratio of the signal is very small, then the SURE method estimation will account for more noises. In this type of situation, the fixed form threshold is selected by means of global thresholding method. *Minimax* (w_{tm}) Minimax threshold is also used fixed threshold and it yields minmax performance for Mean Square Error (MSE) against an ideal procedures.

Table I:Comparison of Previous Wavelet Technique & Proposed PCA Technique interms of PSNR (in dB) for classical *Lena* image distorted by Gaussian Noise of Different Noise Variance .

Variance	Noisy Image	**Proposed PCA**	Rigsure	Heusure	Minimaxi
5	34.14	**35.7**	35.2	35.4	35.61
10	28.11	**29.6**	28.9	29.3	28.9
12	26.49	**28.05**	26.6	27.7	27.25
15	24.64	**26.2**	26.01	25.5	26.02
18	23.03	**24.6**	23.66	23.8	23.48

20	22.14	**23.7**	23.4	22.8	22.5
22	21.34	**23.1**	22.6	21.9	21.7
25	20.22	**21.81**	21.34	20.86	20.5
30	18.68	**19.6**	19.4	19.2	19.1

Table II:Comparison of Previous Wavelet Technique & Proposed PCA Technique interms of PSNR (in dB) for classical *Lena* image distorted by Speckle Noise of Different Noise Variance .

Variance	Noisy Image	**Proposed PCA**	Rigsure	Heusure	Minimaxi
5	39.75	**41.14**	39.78	39.1	40.2d
10	33.73	**35.26**	33.89	35.01	34.7
12	32.22	**33.8**	33.1	33.09	33.09
15	30.25	**31.82**	30.9	31.5	31.07
18	28.69	**30.25**	30.1	29.8	29.4
20	27.76	**29.82**	29.1	28.8	28.4
22	26.9	**28.5**	27.1	27.9	27.5
25	25.81	**27.4**	26.41	26.8	26.4
30	24.25	**25.82**	25.1	25.1	24.7

V. RESULTS

The values of PSNR for various methods are provided in Tables I & II. The proposed Principal Component Analysis method yields the maximum PSNR values and for various noise levels in comparison to the other wavelet thresholding techniques. The denoising results using PCA for classical Lena image provides better performance as compare to other thresholding techniques. Table I gives the comparision of Rigsure ,Heusure,minimaxi thresholding techniques with soft Haar wavelet thresholding with PCA interms of PSNR values.From the values obtained in Table I & II we can infer that the proposed PCA method produces a better result in comparison to other threshold based thresholding techniques .In terms of visual perception and quality metrics PCA method performs well. Very small structures and weak edges are very well preserved in the denoised images obtained from the proposed method.

VI CONCLUSION AND FUTURE WORK

The proposed PCA technique outperforms all the standard filters,lee filter, wavelet soft & hard method. The proposed Method is having improvements in terms of PSNR.PCA method outperforms all other soft thresholding technique. However, by visual inspection it is clear that the denoised image, while removing a large amount of noise, suffers practically no degradation in sharpness and details.Experimental results show that our PCA method yields significantly improved visual quality as well as better PSNR and structure similarity as compared to the other techniques in the denoising literature. The output obtained from proposed method outperforms both Gaussian as well as Speckle Noise.As future work, we would like to work further on the comparison of different denoising techniques. We would also like to reduce mean square error with less processing time.

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